**Refining RFT 4.75 for Cluster Lensing Without Neutrinos**

Refined Relativistic Field Theory (RFT) version 4.75 is being adjusted to fit **cluster gravitational lensing** observations without invoking any unseen neutrino mass component. In particular, we target two critical test cases: the Bullet Cluster (a violent merger showing spatial separation of mass components) and Abell 1689 (a massive, relaxed cluster). By tuning the RFT coupling function f(E,ρ,v)f(E,\rho,v)f(E,ρ,v) – which governs the “resonant” gravitational field’s response to local field energy EEE, density ρ \rhoρ, and relative velocity vvv – we aim to reproduce observed lensing masses and offsets using only the baryonic matter present. Key parameters such as the coupling strength kkk, a critical density ρcrit \rho\_{\rm crit}ρcrit​, and a critical field energy EcritE\_{\rm crit}Ecrit​ will be adjusted within ~20% of their RFT 4.5 baseline values to achieve this. Below we detail the approach for each cluster and the overall execution plan.

**Bullet Cluster Analysis**

**Observational Constraints:** The Bullet Cluster (1E 0657–56) provides a **crucial test** because its hot gas and total mass are spatially segregated after a 4500 km/s cluster collision​

[en.wikipedia.org](https://en.wikipedia.org/wiki/Bullet_Cluster#:~:text=%282004%29%20and%20Clowe%20et%20al,12)

. X-ray observations (Chandra/XMM-Newton) show two clumps of **hot gas (intracluster plasma)** that contain most of the ordinary matter (~10% of the cluster’s mass)​

[astro.rug.nl](https://www.astro.rug.nl/~weygaert/tim1publication/nijmegen06/articles/bulletcluster.clowe.bradac.2006.pdf#:~:text=peaks%20are%20also%20offset%20at,Allen%20et%20al)

. This gas is observed to lag behind the collisionless galaxies due to ram pressure, whereas the **gravitational lensing mass** (traced by a weak lensing κ-map) stays with the galaxies and lies **~200 kpc ahead of the gas**​

[en.wikipedia.org](https://en.wikipedia.org/wiki/Bullet_Cluster#:~:text=%282004%29%20and%20Clowe%20et%20al,12)

. In other words, the lensing mass peaks are roughly coincident with the galaxy concentrations and offset from the X-ray plasma by on the order of a few hundred kpc. The inferred mass components are approximately: **hot gas ~10^14 M⊙**, **stellar mass in galaxies ~10^13 M⊙**, and a **total lensing mass ~10^15 M⊙** for the system​

[arxiv.org](https://arxiv.org/pdf/0704.0094#:~:text=have%20masses%20only%20M1%20%3D,04%20plus%202eV)

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[astro.rug.nl](https://www.astro.rug.nl/~weygaert/tim1publication/nijmegen06/articles/bulletcluster.clowe.bradac.2006.pdf#:~:text=peaks%20are%20also%20offset%20at,Allen%20et%20al)

. Classic analyses (Clowe et al. 2006; Markevitch et al. 2004) report an 8σ significant separation of the lensing mass from the baryonic mass peak, which cannot be explained by modifying gravity alone under MOND-like theories​

[ar5iv.org](https://ar5iv.org/pdf/astro-ph/0608407#:~:text=gravitational%20lensing%20maps%20which%20show,in%20the%20system%20is%20unseen)

. Instead, the Bullet Cluster has been interpreted as direct evidence for dark matter, since most of the mass (≈90%) is **“unseen” collisionless matter** separated from the normal matter​

[chandra.harvard.edu](https://chandra.harvard.edu/graphics/resources/handouts/lithos/bullet_lithos.pdf#:~:text=so,or%20the%20gas%20except%20through)

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[en.wikipedia.org](https://en.wikipedia.org/wiki/Bullet_Cluster#:~:text=%282004%29%20and%20Clowe%20et%20al,12)

. Any alternative theory of gravity must therefore **reproduce the ~200 kpc mass-gas offset and the large mass discrepancy** without added dark matter to remain viable.

**RFT Adjustments (No Neutrinos):** In RFT 4.75, we adjust the function f(E,ρ,v)f(E,\rho,v)f(E,ρ,v) so that the “resonant” gravitational field can mimic a collisionless dark matter component. The goal is to have the RFT gravitational potential **trace the galaxies rather than the gas**, as observed. In RFT 4.5, preliminary fits achieved a mass distribution aligned with the galaxy centroids to within ~20 kpc, but required adding a small neutrino halo (on the order of a few ×10^13 M⊙, ~1–3% of the cluster mass) to boost the lensing signal​

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. We now aim to eliminate this neutrino contribution entirely. To do so, we will **fine-tune parameters** within ~20% of their previous values: for example, increasing the coupling constant kkk (which scales the strength of the field self-interaction) by up to ~20%, and adjusting thresholds like ρcrit \rho\_{\rm crit}ρcrit​ or EcritE\_{\rm crit}Ecrit​ that control when the additional field term becomes significant. These tweaks are designed to enhance the **self-gravity of the field** such that the RFT field can carry inertia and remain with the fast-moving subcluster galaxies, instead of being dragged with the gas. In effect, the RFT field’s stress-energy will behave more like collisionless matter. This should deepen the lensing potential (compensating for the ~9× baryon mass deficit​

[astro.rug.nl](https://www.astro.rug.nl/~weygaert/tim1publication/nijmegen06/articles/bulletcluster.clowe.bradac.2006.pdf#:~:text=peaks%20are%20also%20offset%20at,Allen%20et%20al)

) and **shift the effective mass distribution** to achieve the observed ~200 kpc gas–mass separation **without invoking any neutrino mass**. (By contrast, MOND plus 2 eV neutrinos required a neutrino-to-baryon mass ratio of ~7:1 in this system​

[arxiv.org](https://arxiv.org/pdf/0704.0094#:~:text=need%20for%2C%20e,The)

– a much larger new mass component that RFT seeks to avoid.)

**Parameter Optimization:** We will begin with **analytical approximations** to guide the tuning of f(E,ρ,v)f(E,\rho,v)f(E,ρ,v). The Bullet Cluster can be modeled initially as two point masses (representing the main cluster and subcluster) that have recently collided. Using the known baryonic mass (~1×10^14 M⊙ in gas for the main cluster, ~2–3×10^13 M⊙ for the subcluster) and an assumed relative velocity ~4500 km/s​

[en.wikipedia.org](https://en.wikipedia.org/wiki/Bullet_Cluster#:~:text=%282004%29%20and%20Clowe%20et%20al,12)

, we will write down the modified gravitational potential in RFT. The form of f(E,ρ,v)f(E,\rho,v)f(E,ρ,v) will be adjusted so that, in the instantaneous aftermath of the collision, the potential around each cluster is **dominated by the field’s own contribution rather than the displaced gas**. For example, if fff increases in regions of low density (as in earlier RFT versions) but is also made sensitive to the high relative velocity or shock energy, the subcluster’s fast passage could trigger an enhanced field around the galaxies. We will explore the parameter space near the RFT 4.5 values: e.g. if ρcrit \rho\_{\rm crit}ρcrit​ is ~20% lower, the resonant field might “turn on” more strongly at the Bullet Cluster’s ICM densities (~10^-3 cm^-3), boosting lensing. Likewise, a slight increase in EcritE\_{\rm crit}Ecrit​ (the field energy density threshold) might let the field carry momentum through the collision, maintaining a gravitational mass concentration with the galaxy core. These adjustments will be tested analytically by solving a toy model of the modified Poisson equation for two colliding mass distributions.

**Data Calibration:** Empirical data will constrain these parameters. **Chandra X-ray measurements** provide the gas mass and distribution – roughly 1×10^14 M⊙ of plasma mostly in the main cluster, and ~10% of that in the smaller “bullet” subcluster​

[arxiv.org](https://arxiv.org/pdf/0704.0094#:~:text=have%20masses%20only%20M1%20%3D,04%20plus%202eV)

. The gas profile (from XMM-Newton/Chandra) informs us where the baryonic mass is, and how far it lags behind. We will ensure our model uses a comparable gas density profile and separation (~0.7 Mpc separation between the two cluster centers, with the subcluster gas offset behind its galaxy centroid​

[ar5iv.org](https://ar5iv.org/pdf/astro-ph/0608407#:~:text=The%20cluster%20has%20two%20primary,through%20each%20other%20Myr%20ago)

). The **HST weak lensing map** (Clowe et al. 2006) gives the shape and location of the total mass peaks. It shows the main gravitational potential peaks near the galaxy locations, and an 8σ offset of those peaks from the plasma centers​

[astro.rug.nl](https://www.astro.rug.nl/~weygaert/tim1publication/nijmegen06/articles/bulletcluster.clowe.bradac.2006.pdf#:~:text=Both%20peaks%20are%20offset%20from,Both)

. We will use the observed lensing convergence values to normalize the strength of our resonant field: the RFT gravitational potential depth must produce a similar lensing signal (κ\kappaκ values) as a ~10^15 M⊙ Newtonian mass would. In practice, this means tuning kkk such that the **projected lensing mass** within, say, 250 kpc of the subcluster’s center matches the reported value (~2.3×10^14 M⊙ within 250 kpc for the Bullet subcluster)​

[aanda.org](https://www.aanda.org/articles/aa/full_html/2016/10/aa27959-15/aa27959-15.html#:~:text=known,cluster)

. By matching these profiles in an analytic model, we narrow down the viable f(E,ρ,v)f(E,\rho,v)f(E,ρ,v) forms and parameter sets that can yield the correct **mass partitioning** (≈90% of the gravitational mass in the collisionless component, 10% in gas​

[astro.rug.nl](https://www.astro.rug.nl/~weygaert/tim1publication/nijmegen06/articles/bulletcluster.clowe.bradac.2006.pdf#:~:text=peaks%20are%20also%20offset%20at,Allen%20et%20al)

).

**Simulation Verification:** After the analytical tuning, we will perform **full 3D simulations** of the Bullet Cluster merger using a modified version of *Gadget-4*. The initial conditions will be set to mimic the pre-collision clusters: a main cluster ~1.5×10^15 M⊙ and a subcluster ~2×10^14 M⊙ (consistent with lensing estimates​

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), each containing the observed baryon fractions (gas and galaxies). The RFT modifications (with the optimized f(E,ρ,v)f(E,\rho,v)f(E,ρ,v)) will be implemented in the gravity solver. We will then simulate the cluster collision and track the distribution of the gravitational potential vs. the baryonic matter. The **expected outcome** is that, after core passage, the simulation shows two distinct gravitational mass clumps that remain centered on the galaxy distributions (leading the gas clouds). We will measure the separation between the peak of the lensing potential and the peak of the X-ray gas in the simulation and tune as needed to reach ~200 kpc, in line with observations​

[en.wikipedia.org](https://en.wikipedia.org/wiki/Bullet_Cluster#:~:text=%282004%29%20and%20Clowe%20et%20al,12)

. We will also compare the **total lensing mass** in the simulation to the values inferred from observations (e.g. the surface mass density maps in Clowe et al. 2006): RFT 4.75 should reproduce a combined mass on the order of 10^15 M⊙ within the virial region, apportioned roughly 7:3 between the main and subcluster (since the main cluster contributed ~0.8×10^15 M⊙ and the bullet ~0.3×10^15 M⊙)​

[arxiv.org](https://arxiv.org/pdf/0704.0094#:~:text=Assuming%20Newtonian%20gravity%20the%20models,45%C3%971015M%E2%8A%99)

. Crucially, *no neutrino or dark matter particles* will be included; if the f(E,ρ,v)f(E,\rho,v)f(E,ρ,v) function is successful, the **resonant field itself will supply the necessary additional gravity**. We will iterate on the parameters if needed and ensure that they stay within ~20% of the original values to maintain consistency with galaxy-scale phenomenology from RFT 4.5.

**Abell 1689 Analysis**

**Rationale:** To verify that our RFT refinements are not tuned only to violent mergers, we apply them to **Abell 1689**, a massive but **non-merging** galaxy cluster. Abell 1689 (at *z*≈0.18) is a well-studied, **relaxed cluster** known for its very strong gravitational lensing. Unlike the Bullet Cluster, Abell 1689’s baryonic and lensing mass distributions are centrally aligned (no major recent collision), yet it still exhibits a large mass discrepancy (requiring dark matter in the standard view). This makes it an ideal test of whether RFT 4.75 can account for **cluster-scale dark matter effects in equilibrium**. We will use Abell 1689 to ensure the modified f(E,ρ,v)f(E,\rho,v)f(E,ρ,v) works in the absence of high relative velocities – the field must still augment gravity enough to explain lensing, purely from the cluster’s static configuration.

**Observational Inputs:** Chandra X-ray observations of Abell 1689 provide the radial gas mass profile and total X-ray mass estimates under hydrostatic equilibrium. The intracluster **gas mass is on the order of 10^14 M⊙**, constituting roughly 10% of the cluster’s total mass within ~1 Mpc​

[researchgate.net](https://www.researchgate.net/publication/273157741_Three-dimensional_Multi-probe_Analysis_of_the_Galaxy_Cluster_A1689#:~:text=predicted%20distribution,for%20lensing%20projection%20effects%2C%20respectively)

. (For example, within a 0.9 Mpc radius, the gas mass fraction is reported as ~10%​

[researchgate.net](https://www.researchgate.net/publication/273157741_Three-dimensional_Multi-probe_Analysis_of_the_Galaxy_Cluster_A1689#:~:text=predicted%20distribution,for%20lensing%20projection%20effects%2C%20respectively)

.) The mass in stars (galaxies) is smaller, a few percent of the total. Gravitational lensing data – including **HST strong lensing** and Subaru weak lensing – indicate a **total mass of ~1–2×10^15 M⊙** for Abell 1689 within its virial radius​

[arxiv.org](https://arxiv.org/abs/1005.0398#:~:text=structures%20with%20some%20slight%20deviations,1.2)

. Notably, Abell 1689 has an Einstein radius of ~47″ (≈140–150 kpc) for background sources at *z*~2​

[arxiv.org](https://arxiv.org/abs/1005.0398#:~:text=images%20of%2055%20knots%20residing,15%20Msun%20%2F%20h70)

, one of the largest known, signaling a very deep potential well. Any successful RFT model must reproduce the high central lensing convergence (κ) observed, given the modest baryonic mass present. We will use the **lensing mass profile from HST/ACS** (e.g. the mass enclosed vs radius from strong+weak lensing analyses by Coe et al. 2010) as the target for our RFT predictions, and the **Chandra gas profile** (e.g. temperature and density distribution from X-ray) as the source for baryonic mass input.

**Applying RFT 4.75:** We will apply the same f(E,ρ,v)f(E,\rho,v)f(E,ρ,v) function optimized in the Bullet Cluster case directly to Abell 1689. In this relaxed cluster, the relative velocities vvv are low and no recent shock is present, so the velocity-dependent aspects of f(E,ρ,v)f(E,\rho,v)f(E,ρ,v) (if any) should have minimal effect – the function’s behavior will be dominated by the density/field strength terms f(E,ρ)f(E,\rho)f(E,ρ). The **critical test** is whether the resonant field can amplify gravity in a static scenario enough to account for the factor ~10 discrepancy between baryonic mass and lensing mass. Using the observed gas density profile as input, we will solve the modified Poisson equation (either analytically in spherical symmetry or via a hydrostatic equilibrium code) to find the RFT gravitational potential profile. The parameters k,ρcrit,Ecritk, \rho\_{\rm crit}, E\_{\rm crit}k,ρcrit​,Ecrit​ tuned for the Bullet Cluster will be held fixed (or adjusted very slightly, if absolutely needed within the 20% range). We expect that if RFT’s extra field term is effective, it will **act like an additional mass distribution**. For instance, at radii ~0.5–1 Mpc, where the cumulative gas mass ~few×10^13 M⊙ is far too low to explain the lensing (~5×10^14 M⊙ within 1 Mpc), the RFT field should contribute an equivalent of several ×10^14 M⊙ of gravitational mass. We will look for a solution where the **modeled lensing convergence profile matches the HST observations** (Coe et al. and others) within uncertainties, without inserting dark matter. In essence, the RFT field around Abell 1689 must **mimic an NFW-like dark matter halo** of mass ~1e15 M⊙ and concentration ~c≈10​

[arxiv.org](https://arxiv.org/abs/1005.0398#:~:text=kpc,Our)

purely through the modified field equations.

**Testing and Validation:** As with the Bullet Cluster, we will perform a simulation or equilibrium test. For Abell 1689, a full cosmological simulation is not strictly needed since it’s a static system; instead, we can take an equilibrium approach: start with the observed gas density profile (and an assumed distribution of galaxies), then compute the gravitational potential by solving ∇²Φ = 4πG(ρ\_baryon + ρ\_field(Φ,ρ)). Here ρ\_field is an effective density due to RFT’s extra term. We will iterate this solution until convergence (ensuring the gas is supported by the pressure gradient consistent with the potential, if we want to be self-consistent). We will verify that the **projected mass profile** (integrating the modeled Φ or κ) matches the **weak lensing mass profile** from observations​

[arxiv.org](https://arxiv.org/abs/1005.0398#:~:text=structures%20with%20some%20slight%20deviations,1.2)

. A successful fit means RFT 4.75 can **hold up for a relaxed cluster** – i.e. the theory does not rely on dramatic dynamics like the Bullet collision to explain missing mass. Additionally, we will confirm that **no “hidden” mass** (no neutrinos, MACHOs, etc.) is required: the gas and galaxies we put in are all the matter present. If the lensing still comes out low in the model, that would indicate a failure of RFT in this regime (similar to how MOND under-predicts cluster lenses without dark matter​

[arxiv.org](https://arxiv.org/pdf/0704.0094#:~:text=have%20masses%20only%20M1%20%3D,04%20plus%202eV)

). But our expectation is that the tuned RFT field will fill in the gap. For example, if in RFT the vacuum (or low-density) regions inside the cluster contribute an extra acceleration, the mass deficit can be remedied by the field’s self-energy. We will compare the **gas mass fraction** in the RFT solution to expectations: observations show ~10% gas fraction​

[researchgate.net](https://www.researchgate.net/publication/273157741_Three-dimensional_Multi-probe_Analysis_of_the_Galaxy_Cluster_A1689#:~:text=predicted%20distribution,for%20lensing%20projection%20effects%2C%20respectively)

, and with RFT we effectively want the rest (~90%) of gravity to come from the field. This test will cement whether the chosen f(E,ρ,v)f(E,\rho,v)f(E,ρ,v) yields a universal improvement or was over-specialized to merging situations.

**Execution Plan**

To summarize, the refinement of RFT 4.75 will be carried out in a staged approach:

1. **Analytical Modeling (Bullet Cluster):** Develop a simplified model of the Bullet Cluster merger to understand how changes in f(E,ρ,v)f(E,\rho,v)f(E,ρ,v) affect the lensing mass distribution. Tune the RFT parameters (k,ρcrit,Ecritk, \rho\_{\rm crit}, E\_{\rm crit}k,ρcrit​,Ecrit​, etc.) within ~20% of their prior values to reproduce the observed ~200 kpc offset and the required lensing mass (~10^15 M⊙) with only baryonic matter​

[en.wikipedia.org](https://en.wikipedia.org/wiki/Bullet_Cluster#:~:text=%282004%29%20and%20Clowe%20et%20al,12)

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[astro.rug.nl](https://www.astro.rug.nl/~weygaert/tim1publication/nijmegen06/articles/bulletcluster.clowe.bradac.2006.pdf#:~:text=peaks%20are%20also%20offset%20at,Allen%20et%20al)

. Use published data (gas mass from X-ray, total mass from lensing) as calibration points in this tuning​

[arxiv.org](https://arxiv.org/pdf/0704.0094#:~:text=have%20masses%20only%20M1%20%3D,04%20plus%202eV)

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[aanda.org](https://www.aanda.org/articles/aa/full_html/2016/10/aa27959-15/aa27959-15.html#:~:text=known,cluster)

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1. **Cross-Check with Observations:** Compare the analytically optimized model to empirical Bullet Cluster observations. Ensure the **lensing peak aligns with the galaxy centroid** (not the gas) and the magnitude of the lensing convergence matches the values from weak lensing maps​

[astro.rug.nl](https://www.astro.rug.nl/~weygaert/tim1publication/nijmegen06/articles/bulletcluster.clowe.bradac.2006.pdf#:~:text=peaks%20are%20also%20offset%20at,Allen%20et%20al)

. Adjust the function f(E,ρ,v)f(E,\rho,v)f(E,ρ,v) form iteratively until the model’s predictions (mass split, offset, required field contribution) are consistent with Clowe et al. 2006 and Markevitch et al. 2004 constraints. Pay special attention that this is achieved **without invoking neutrinos** – the model should explain the data purely through the modified field​

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1. **Gadget-4 Simulation (Bullet Cluster):** Implement the refined f(E,ρ,v)f(E,\rho,v)f(E,ρ,v) in an $N$-body/hydrodynamics code (Gadget-4) and run a detailed Bullet Cluster collision simulation. Start from pre-merger initial conditions (two clusters with masses and baryon content as observed) and evolve through the merger. **Verify the outcomes:** the separation between gas and lensing mass ~200 kpc is reproduced, and the total lensing mass profile fits the observed weak lensing map​

[en.wikipedia.org](https://en.wikipedia.org/wiki/Bullet_Cluster#:~:text=%282004%29%20and%20Clowe%20et%20al,12)

. Also check that the **mass estimates from a lensing perspective** (fitting the simulated $\kappa$-map as if it were data) are ~1×10^15 M⊙ without any dark mass added – i.e., RFT’s field is providing the extra gravity that a conventional analysis would attribute to dark matter​

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1. **Apply to Abell 1689 (Analytical):** Take the tuned RFT parameters and apply them to a static model of Abell 1689. Using the cluster’s gas density profile from Chandra and assuming the galaxies trace the dark matter light distribution, solve for the gravitational potential under RFT. **Check the mass distribution:** the model should yield a total mass of order 1–2×10^15 M⊙ within the virial radius​

[arxiv.org](https://arxiv.org/abs/1005.0398#:~:text=kpc,1.2)

, matching lensing observations, while the input baryons are only ~2×10^14 M⊙ (gas + stars)​

[researchgate.net](https://www.researchgate.net/publication/273157741_Three-dimensional_Multi-probe_Analysis_of_the_Galaxy_Cluster_A1689#:~:text=predicted%20distribution,for%20lensing%20projection%20effects%2C%20respectively)

. No additional unseen mass is to be included. If needed, make minor tweaks to f(E,ρ,v)f(E,\rho,v)f(E,ρ,v) (within allowed bounds) to improve the fit, but **preferably use the same function from the Bullet Cluster** to demonstrate consistency.

1. **Simulation/Validation (Abell 1689):** If feasible, run a simulation or equilibrium solver for Abell 1689 under RFT. This could involve letting an isolated cluster (with mass and gas profile as observed) relax in the Gadget-4 code with the RFT modifications. Confirm that the **gravitational lensing signal (e.g. deflection angles or projected mass)** in the RFT run matches the observed weak lensing profile from HST (e.g. the Einstein radius ~140 kpc and high central convergence)​

[arxiv.org](https://arxiv.org/abs/1005.0398#:~:text=images%20of%2055%20knots%20residing,15%20Msun%20%2F%20h70)

. Because Abell 1689 is relaxed, verify that RFT doesn’t produce any spurious offsets or asymmetries – the field-enhanced gravity should simply deepen the potential well symmetrically.

1. **Report Findings & Next Steps:** Document the optimized parameter values and how the new RFT 4.75 explains both the Bullet Cluster and Abell 1689 observations. Highlight that **no neutrino or dark matter component is required** in these fits, a significant improvement over RFT 4.5 (which needed a ~1% mass neutrino tweak) and over MOND-like theories (which require additional mass like 2 eV neutrinos for clusters)​

[arxiv.org](https://arxiv.org/pdf/0704.0094#:~:text=need%20for%2C%20e,The)

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. Once the cluster-scale tests are successful, proceed to the next stage of RFT development: testing the theory against cosmological data (e.g. the cosmic microwave background power spectrum and large-scale structure formation). Ensuring consistency with CMB and structure formation will solidify RFT 4.75 as a viable alternative to dark matter across all scales. The cluster results will serve as crucial evidence that RFT can **match gravity on Mpc scales** without unseen mass, bridging the gap between galaxy-scale successes and cosmological predictions.